1. The functions f(x) and g(x) are defined by $f(x) = \ln x$ and $g(x) = 2 + e^x$, for $x > 0$.

Find the exact value of x, given that $fg(x) = 2x$.

[5]

2. In an experiment, the temperature of a hot liquid is measured every minute. The difference between the temperature of the hot liquid and room temperature is D °C at time t minutes.

Fig. 8 shows the experimental data.

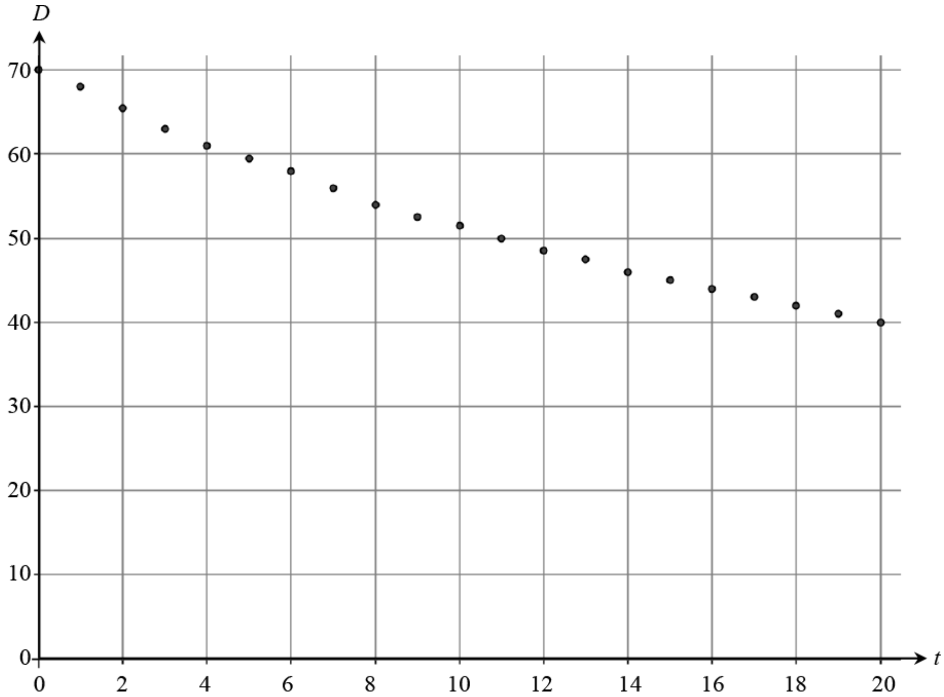


Fig. 8

It is thought that the model $D = 70e^{-0.03t}$ might fit the data.

(a) Write down the derivative of $e^{-0.03t}$. [1]

(b) Explain how you know that $70e^{-0.03t}$ is a decreasing function of t. [1]

(c) Calculate the value of $70e^{-0.03t}$ when

(i) $t = 0$, [1]

(ii) $t = 20$. [1]

(d) Using your answers to parts (b) and (c), discuss how well the model $D = 70e^{-0.03t}$ fits the data. [3]

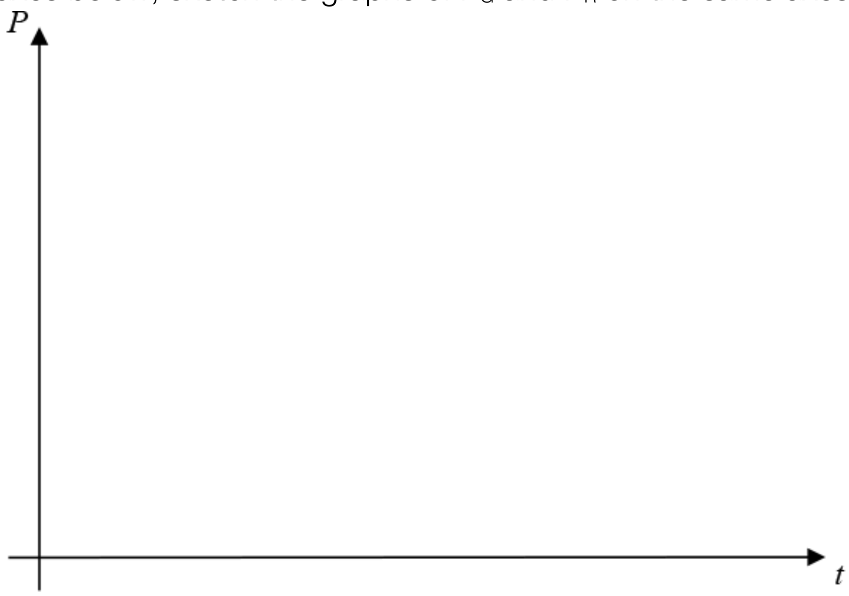
3. In a certain region, the populations of grey squirrels, P_G and red squirrels P_R, at time t years are modelled by the equations:

$$P_G = 10000(1 - e^{-kt})$$

$$P_R = 20000e^{-kt}$$

where $t \geq 0$ and k is a positive constant.

(a) (i) On the axes below, sketch the graphs of P_G and P_R on the same axes.



(ii) Give the equations of any asymptotes. [4]

(b) What does the model predict about the long term population of

- grey squirrels
- red squirrels?

[2]

Grey squirrels and red squirrels compete for food and space. Grey squirrels are larger and more successful than red squirrels.

(c) Comment on the validity of the model given by the equations, giving a reason for your answer. [1]

(d) Show that, according to the model, the rate of decrease of the population of red squirrels is always double the rate of increase of the population of grey squirrels. [4]

(e) When $t = 3$, the numbers of grey and red squirrels are equal. Find the value of k. [4]

4. (a) Sketch the curve $y = e^{2x}$. [2]

(b) Describe fully the transformation that maps the curve $y = e^{x}$ onto the curve $y = e^{2x}$. [2]

(c) Find the equation of the tangent to $y = e^{2x}$ at the point where $x = 3$, giving your answer in the form [6]
$y = e^{a}(bx + c)$ where a, b and c are integers.

5. Fig. 3 shows the curve with equation $y = \sqrt{1 + \ln x}$.

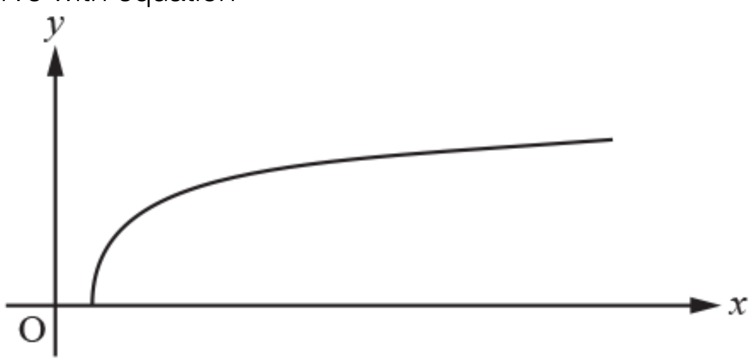


Fig. 3

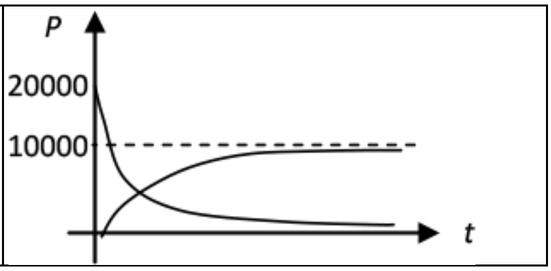
Determine the exact x-coordinate of the point where the curve meets the x-axis. [2]

END OF QUESTION paper

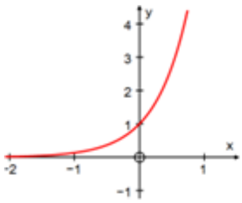
Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1			fg(x) = ln(2 + e^x)	M1	condone missing brackets	
			⇒ ln(2 + e^x) = 2x			
			⇒ 2 + e^x = e^{2x}	A1		
			⇒ $e^{2x} - e^x - 2$ [= 0]	M1	Rearranging into a quadratic in e^x	may be implied from both correct roots
			⇒ (e^x − 2)(e^x + 1) = 0, e^x = 2, −1	A1	obtaining roots 2, −1	−1 root may be inferred from factorising
			⇒ e^x = 2, x = ln 2	A1	x = ln 2 only, not from ww	x = ln (−1) is A0 **Examiner's Comments** Virtually all candidates formed the composite function in the correct order to obtain fg(x) = ln(2 + e^x). A few then simplified this to ln2 + x and therefore made no further progress. Of those who did correctly proceed to 2 + e^x = e^{2x}, a substantial minority then incorrectly took logs of each side to reach ln2 + x = 2x. Of those who correctly rearranged the equation into a quadratic in e^x, nearly all then gained full marks, correctly rejecting the e^x = −1 solution.
			Total	5		

2		a	$-0.03e^{-0.03t}$	B1(AO1.2) [1]		
		b	Decreasing function because $e^{-0.03t}$ is positive [for all values of t] so the gradient is negative.	E1(AO2.2a) [1]	Explanation may include a sketch graph of the function $70e^{-0.03t}$ but it must be clear that the graph is of the function and the answer must clearly refer to the gradient of the function and not the trend in the data	
		c	**A** 70	B1(AO1.1) [1]		
		c	**B** 38.[4168…]	B1(AO1.1) [1]		
		d	Data values decreasing so decreasing function is suitable	E1(AO3.5a) B1(AO3.5a) B1(AO3.5b)		

			At $t = 0$, calculated D = 70 and this matches the data At $t = 20$, data value is 40 which is not exact but close	[3]		
			Total	7		
3		a	A 	M1(AO1.1) M1(AO1.1)	P_G shape through O P_R shape through (0, 20000), [condone graphs for –ve t]	
		a	B asymptote for P_G = 10 000 Asymptote for P_R = 0	A1(AO2.2a) A1(AO2.2a) [4]	Or p = 10 000 Or p = 0	
		b	Red squirrels zero Grey 10 000	B1(AO3.4) B1(AO3.4) [2]		
		c	One relevant comment evaluating the validity of the model	B1(AO3.5a)	E.g. One of • Grey population increases as would be expected [since grey squirrels are larger and more successful] • Red population decreases as would be expected [since red squirrels	

				[1]	have to compete with the larger grey squirrels for food] • Number of squirrels tends to a limit as would be expected [since there is limited food and space] • Would expect grey population to grow slower at first • Would expect red population to fall slower at first	
		d	$\frac{dP_G}{dt} = 10\ 000ke^{-kt}$ $\frac{dP_R}{dt} = -20\ 000ke^{-kt}$ so $\frac{dP_R}{dt} = -2\frac{dP_G}{dt}$	M1(AO3.1b) A1(AO1.1) A1(AO1.1) E1(AO2.1) [4]	Attempts to differentiate either or both Or in words	

		e	10 000 (1 – e^{-3k}) = 20 000 e^{-3k} ⇒ 1 – e^{-3k} = 2 e^{-3k} $\Rightarrow e^{-3k} = \frac{1}{3}$ $\Rightarrow -3k = \ln\left(\frac{1}{3}\right)$ $\Rightarrow k = -\frac{1}{3}\ln\left(\frac{1}{3}\right) = 0.366 \text{ or } \frac{1}{3}\ln 3$	M1(AO1.1a) A1(AO1.1) M1(AO1.1) A1cao(AO2.1) [4]	Taking natural logs of both sides	
			Total	15		
4		a		G1(AO 1.2) G1(AO 1.2) [2]	Correct shape with *x*-axis as asymptote (0, 1) clearly shown	
		b	Stretch in the *x*-direction Stretch scale factor $\frac{1}{2}$	B1(AO 1.1a) B1(AO 1.1a) [2]	'Stretch' must be seen at least once for any marks to be awarded, but the word needn't be repeated	
		c	$\frac{dy}{dx} = 2e^{2x}$	M1(AO 1.1a) A1(AO 1.1b) A1(AO 1.1a)	Allow for any ke^{2x}	

			When $x=3,\ \frac{dy}{dx}=2e^6$ and $y = e^6$ Tangent is $y - e^6 = 2e^6(x - 3)$ $y = 2e^6x - 5e^6$ $y = e^6(2x - 5)$	B1(AO 1.1a) M1(AO 1.1a) A1(AO 2.1) [6]	Correct $2e^{2x}$ Allow method if 403 or better used for e^6 Must be in this form	
			Total	10		
5			$\sqrt{1+\ln x}=0 \Rightarrow \ln x=-1$ $x=\frac{1}{e}$	M1(AO1.1a) A1(AO2.2a) [2]	oe but must be exact value	
			Total	2		